Astro 310
Fall 2001

Problem Set 2 - Solutions

I. Stellar Masses and Radii from an Eclipsing Binary

1) Balancing centripetal and gravitational forces for star 1:

$$\mathbf{M_1\, v_1^2/a_1 = GM_1M_2/(a_1 + a_2)^2} \qquad \text{(a)}$$

(note that centripetal "force" has to be written relative to the center of mass) For circular orbits (Figure 1),

$$\mathbf{v_1 = 2\pi a_1/P} \qquad \text{(b)}$$

So $4\pi^2 a_1/P^2 = GM_2/(a_1 + a_2)^2$ (c)

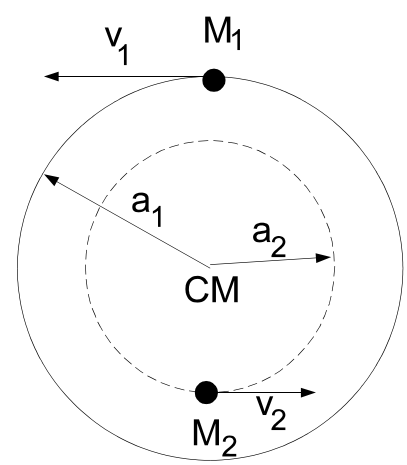


Figure 1

From definition of center of mass,

$$\mathbf{a_1M_1 = a_2M_2}$$

so
$$a \equiv a_1 + a_2 = a_1\,(1 + a_2/a_1)$$
$$= a_1\,(1 + M_1/M_2)$$
$$= a_1/M_2\,(M_1 + M_2) \qquad \text{(d)}$$

Putting (d) in (c)

$$(4\pi^2/P^2)\,(a_1 + a_2)/(M_1 + M_2) = G/(a_1 + a_2)^2$$

And finally (Newton's generalization of) Kepler's IIIrd:

$$\mathbf{a^3/P^2 = (G/4\pi^2)\,(M_1 + M_2)}$$

Remember in units of the Earth-Sun orbit, a in AU, P in years, and M in $M_\odot$,

$$a(AU)^3/P(yr)^2 = (M_1 + M_2)/M_\odot$$

To get Kepler's third with velocities instead of orbital sizes, note

$$v_1 + v_2 = (2\pi/P)\,(a_1 + a_2),$$

So $\mathbf{(v_1 + v_2)^3 = (2\pi G/P)\,(M_1 + M_2)}$

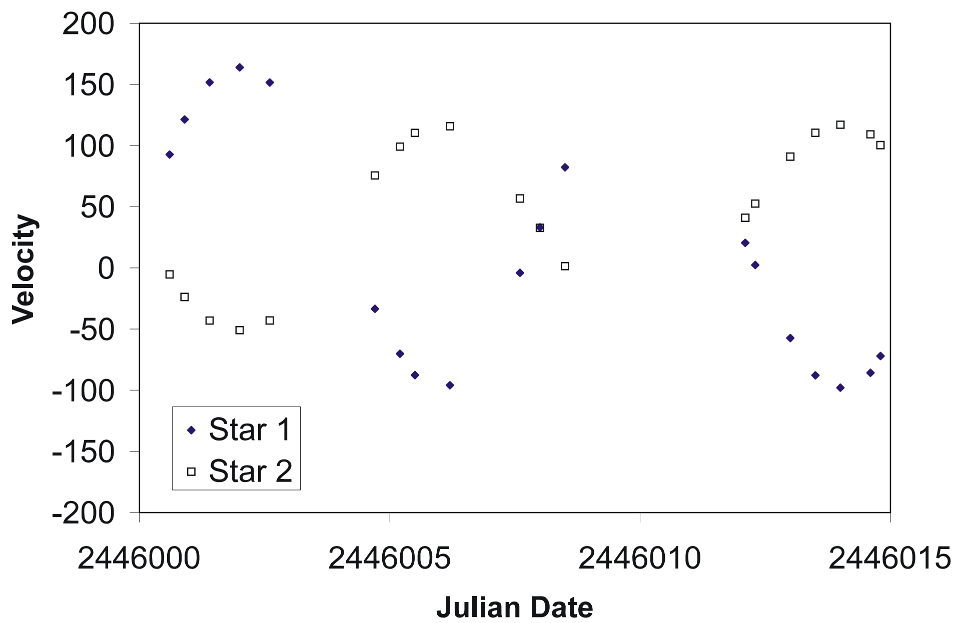
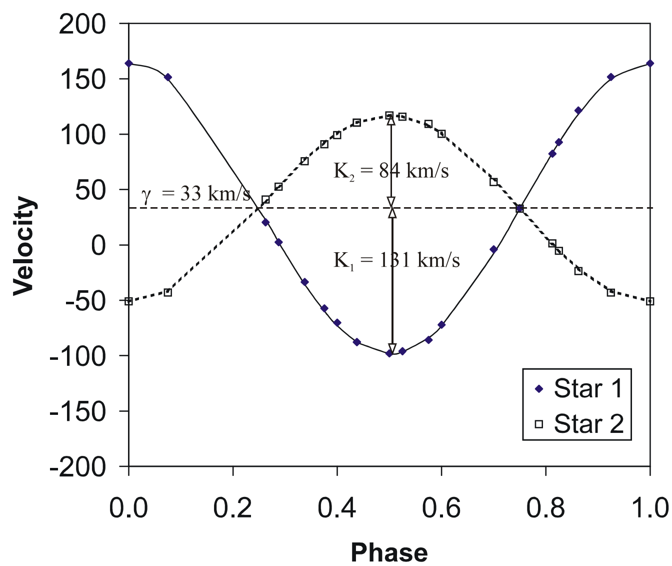


Figure 2

2) First plot up the data vs time (Fig 2). First, one must figure out which star goes with which velocity, by trying to find smooth curves. Next, by inspection find the period:

P = 8 days = 0.022 years

For example, there is a completely covered quarter period (i.e., maximum velocity to crossover) from days 12 to 14. Alternatively, there are two minima at days 6 and 14. Or there is a maximum and minimum (i.e., half period) at days 2 and 6. All of these clues lead to P = 8 days. Of course, the best test is to use a candidate period (e.g., 8 days) to plot the radial-velocity data against orbital phase, <u>all folded into one cycle (plot the phase modulo 1.0)</u>. In this simple case, producing such a phase diagram just requires shifting on Figure 1 the velocities in the second cycle by one period (i.e., 8 days) and rescaling the x-axis from 0-8 days to 0-1 (or 0-2π) in phase (Figure 3). If the candidate period is correct, all of the data will lie on well-defined radial-velocity curves, as they do for a period of 8 days. (This technique for finding a period may not seem sophisticated, but its not far different from standard practice, at least for a first estimate. Computing the rest of the orbital elements is usually done in a more elegant fashion.)



The orbit is sinusoidal (symmetric about the velocity maxima and about the crossovers) indicating a circular orbit or e = 0.0. For a circular orbit symmetry arguments show that V_0 or the γ velocity is just

$$\boldsymbol{\gamma = (v_{max} + v_{min})/2 = 33 \text{ km/sec}}$$

(using the velocities of either component). Alternatively, for any eccentricity the γ velocity of a double-line spectroscopic binary is the velocity of the crossover point in the orbit, so the day 8 observation also tells you γ = 33 km/sec. In addition,

$$\boldsymbol{K_1 = (v_{1,max} - v_{1,min})/2 = 131 \text{ km/sec}}$$

and $\boldsymbol{K_2 = (v_{2,max} - v_{2,min})/2 = 84 \text{ km/sec}}$.

3.) From lecture notes, for non-inclined circular orbits

$$K_{1,2} = 2\pi\, a_{1,2} \sin i/(P(1\text{-}e^2)^{1/2})$$

$$= 2\pi\, a_{1,2}\,/P \;= v_{1,2} \qquad \text{for i=90, e=0.}$$

So $\boldsymbol{a = a_1 + a_2 = (P/2\pi)\,(K_1 + K_2)}$

$$= (0.022 \text{ yr}/2\pi)\,(215 \text{ km/sec}) / (4.74 \text{ (km/sec)/(AU/yr)})$$

$$\boldsymbol{= 0.159 \text{ AU}}$$

So from Kepler's IIIrd (note how easy this is with a in AU and P in years!)

$$\boldsymbol{M_1 + M_2} = .16^3/.022^2 = \boldsymbol{8.3\ M_\odot}$$

From problem 1

$$\boldsymbol{M_1/M_2 = a_2/a_1 = K_2/K_1 = 0.641}$$

$$\boldsymbol{M_1} = (M_1 + M_2) \times 0.641/1.641 = \boldsymbol{3.24\ M_\odot}$$

$$\boldsymbol{M_2} = 3.24/0.641 = \boldsymbol{5.06\ M_\odot}$$

4.) See lecture notes for one formulation of the relevant formulae:

For the smaller star $\quad R(small)/a = \pi\,(t_2 - t_1)/P$
For the larger $\quad R(large)/a = \pi\,(t_4 - t_2)/P$

From the light curve, t_1 = (phase of first contact) × period = 0.16 × 8 days = 1.25 days, t_2 = 1.5 days, t_3 = 2.5 days and t_4 = 2.75 days. Thus, inserting these numbers in the formulae,

$\mathbf{R(small)} = 0.159 \text{ AU} \times \pi \times 0.25 \text{ da} / 8 \text{ da}$
$= 0.0156 \text{ AU} = \mathbf{3.36\ R_\odot}$

$\mathbf{R(large)} = 0.159 \text{ AU} \times \pi \times 1.25 \text{ da} / 8 \text{ da}$
$= \mathbf{16.8\ R_\odot}$

5.) The key to this question is appreciating that whichever star is being eclipsed, the same surface area is always being covered. When the smaller star is in eclipse, its observable surface area A is entirely covered. When the larger star is in eclipse, again only a fraction of that star equaling the observable surface area of the smaller star A is being covered. Thus the amounts of light lost in the two cases

$\Delta L_1 = A\sigma T_{eff,1}^4$
and $\Delta L_2 = A\sigma T_{eff,2}^4$

Thus the ratio of the depths of the eclipses is the ratio of T_{eff}^4. In this problem the ratio of the depths of the eclipses is 30:1 so the ratio of the effective temperatures

$$\mathbf{T_{eff1}/T_{eff2} = 30^{1/4} = 2.34}$$

Note that this procedure is only valid if one has a bolometric light curve!

6.) Since one star is a giant from the spectral type, it is likely that it is the larger and cooler star of the two, and that during primary eclipse the small hotter star is totally eclipsed by the giant. Since the ratio of temperatures is 2.34:1 and a K0III star has a temperature of 4360 K, the temperature of the other star is 4360 × 2.34 = 10700 K or spectral type B9. Since it is the smaller star, it must be a B9V or perhaps a B9IV. To compute the luminosities,

$\mathbf{L(giant)} = (R/R_\odot)^2 \times (T_{eff}/5770)^4$
$= \mathbf{90\ L_\odot}$

and $\mathbf{L(ms) = 129\ L_\odot}$.

Finally, since the giant is more evolved it must be the more massive star. So we have

	Star 2	**Star 1**
M	**5.0 $M_\odot$**	**3.2 $M_\odot$**
Type	**K0III**	**B9V**
T_{eff}	**4360 °K**	**10700 °K**
R	**16.6 $R_\odot$**	**3.3 $R_\odot$**
L	**94 $L_\odot$**	**129 $L_\odot$**

Which is a lot of precise information from one binary!

II. Spectral Types

The star numbers on the unknowns are actually HD numbers. Almost all of these stars are in the "Yale Bright Star Catalogue" (see catalogue shelf), and have very good MK types. As many of you noticed, this is an unrepresentative collection; we need some more cool stars and giants. Here, for your amusement, are the "correct" answers:

HD	Name	V	Spectral Type
1280	θ And	4.6	A2V
4614	η Cas	3.4	G0V+dM0 (a binary)
6961	θ Cas	4.3	A7V
8538	δ Cas	2.7	A5III-IV
8965	-	7.3	B5
10307	HR 483	5.0	G1.5V
14633	-	7.7	B
16895	θ Per	4.1	F8V
20630	κ^1 Cet	4.8	G5V
23338	19 Tau	4.3	B6IV
24131	HR 1191	5.8	B1V
26690	46 Tau	5.3	F2V+F5V (another binary)
34078	AE Aur	6.0	O9.5Ve (a variable emission-line star)
56537	λ Gem 3.6	A3V	
58946	ρ Gem 4.2	F0V	
97603	δ Leo	2.6	A4V
101501	61 UMa	5.3	G8V
103287	γ UMa	2.4	A0Ve
113139	78 UMa	4.9	F2V
115043	-	6.8	G1V
173667	110 Her	4.2	F6V
207538	-	6.9	B3
210027	ι Peg	3.8	F5V
214680	10 Lac 4.9	O9V	
222173	ι And	4.3	B8V
222661	ω^2 Aqr	4.5	B9.5V